

# MULTIVARIATE ANALYSIS FOR COMPARING DIFFERENT METHODS FOR TESTING AGGREGATES

Jürgen PILZ<sup>1</sup>, Gunnar BJARNASON<sup>2</sup> and Pétur PÉTURSSON<sup>3</sup>

<sup>1</sup>Professor of Applied Statistics, University of Klagenfurt, Austria

<sup>2</sup>Chief of Geological Department, Public Roads Administration, Iceland

<sup>3</sup>Road Research Division, Icelandic Building Research Institute

## 1. INTRODUCTION

A multivariate data analysis of 20 Icelandic base course aggregate samples taken in the year 1995 is presented. The sample locations and the geological background have been described in Bjarnason et al. (1999). The samples had been investigated by various degradation tests, which can be grouped into three major classes: fragmentation tests, durability tests and abrasion tests. The following fragmentation test methods were used: Los Angeles (LA) test, the German Schlagversuch (SZ) test, standard and modified Proctor Bg-index tests, the British Ten per cent Fines Value tests (TFVdry and TFVwet), the Aggregate Impact Value tests (AIVdry and AIVwet) and the Dutch static compression tests (DSC5.0 and DSC1.6). With regard to durability, weathering and abrasion (DWA), the following test methods were considered: Icelandic Freeze/Thaw test, Freeze/Thaw Index, Nordic Freeze/Thaw test (Nordtest) and Freeze/Thaw EN1367-1, Magnesium Sulphate Soundness (MSS) test, Methylene Blue Absorption test, Studded Tyre (ST) test and Micro-Deval test. Test results were compared with alteration and porosity of the aggregates according to petrographic analysis.

Although the LA test is often called LA abrasion test, Bjarnason et al. (1999) argue that it gives results which are much more like those of the fragmentation test methods. This was confirmed by the rather high (positive) correlation values and the results of a factor analysis, where the LA test method was grouped together with the fragmentation tests, being located far away from and almost orthogonal to the abrasion tests. The interpretation of the results, especially of a factor analysis, is however, much more complicated.

## 2. PRINCIPAL COMPONENT ANALYSIS

This paper reports results from a more detailed study and comparison of the test methods, using principal component analysis (PCA) as an additional tool and then proceeding with factor analysis to define a statistically sound grouping. The statistical software system R<sup>®</sup> and the software PM-Techmodel 1.0<sup>®</sup> were used, the latter one has recently been developed by Petromodel Ltd., in cooperation with the Steinbeis Transfer Centre of Statistical Data Analysis at the University of Klagenfurt, Austria.

First a principal component analysis (PCA) was performed, separately for the 10 fragmentation test variables and the 10 durability and abrasion test variables. PCA is a multivariate analysis technique, which aims at the reduction of dimensionality, by transforming the original variables to a new set of uncorrelated principal components, which are ordered according to the percentage of data variability they explain.

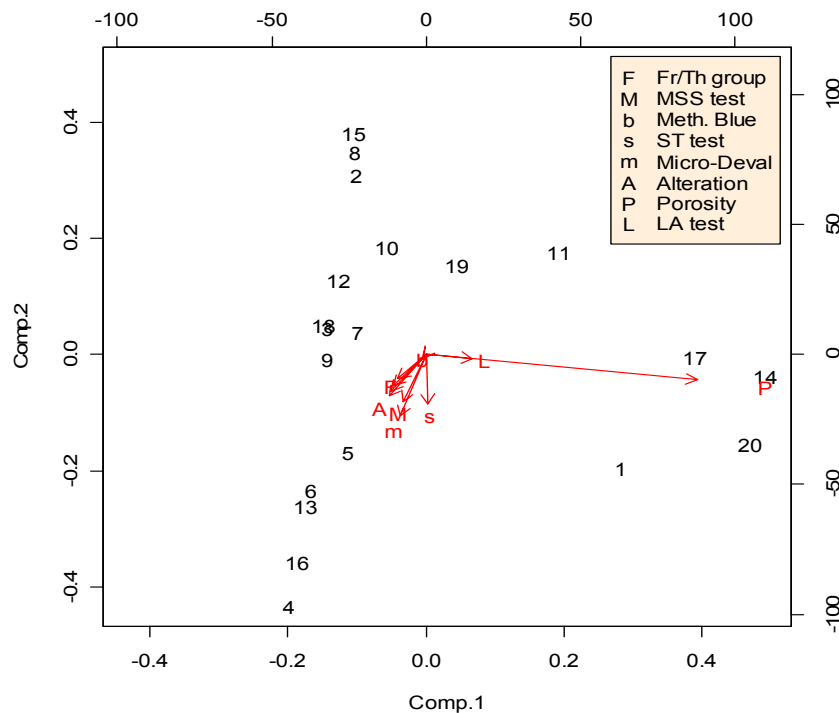
The PCA for the 10 fragmentation test methods shows that the first two components explain nearly 90% of the variability, and with four components more than 95% of data variability is explained. For the 10 durability and abrasion test variables the explanation is not as strong, but still convincing (the first 2 components explain 80% and the first 4 components explain more than 94%). This suggests that the durability and weathering tests are not as homogeneous as the fragmentation tests.

### 3. FACTOR ANALYSIS AND BILOTS

In PCA, the observed variables are the quantities of interest. In factor analysis (FA), by contrast, the observed variables are of relatively little intrinsic interest – the underlying factors are the quantity of interest. Essentially, the covariance matrix  $C$  of the data is decomposed into a factor covariance matrix  $F=LL^T$ , where  $L$  is the so-called matrix of *factor loadings*, and a diagonal error covariance matrix  $U$ , the elements of which are called *uniqueness coefficients*. Whereas PCA does not require distributional assumptions, FA requires approximate multivariate normality of the data to estimate the components in  $L$  and  $U$  and for testing whether a given number of factors is sufficient.

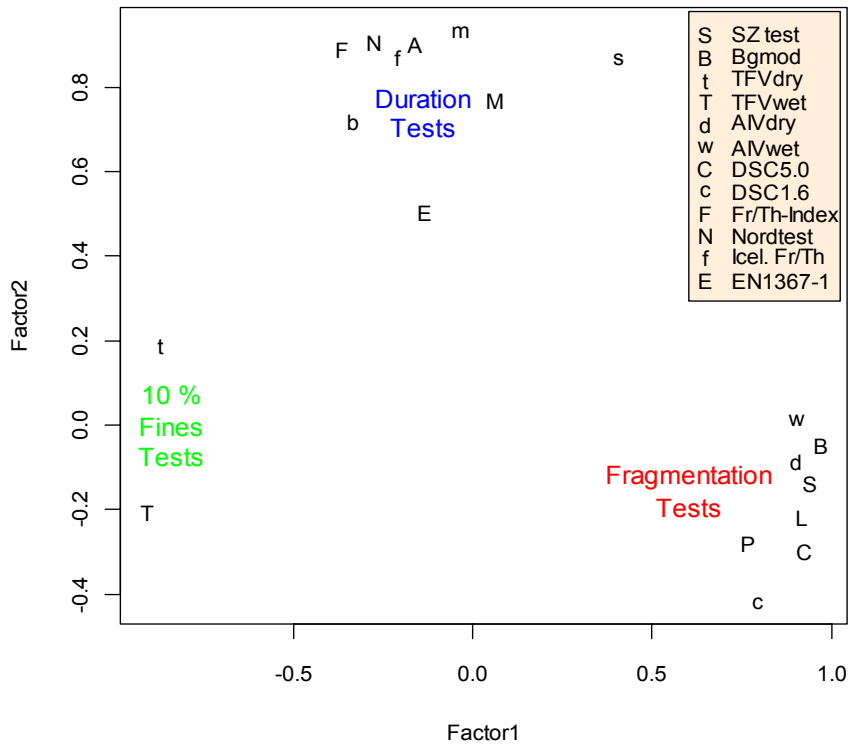
When investigating the fragmentation tests alone, the surprising result is that only one factor is sufficient (p-value = 0.09), i.e. these can all be summarised in a single fragmentation test method group. The separate factor analysis of the durability and abrasion test methods reveals that even 2 factors are not sufficient, the p-value being 0.0197. The null hypothesis of 3 factors is accepted, the p-value being 0.145. The maximum number of test methods, which form a single homogeneous factor, is 7 and includes the 4 different freeze/thaw tests plus the MSS, Micro-Deval and Alteration variables. The porosity variable always forms a specific factor of its own.

A good way for representing the results of PCA and FA, respectively, is the so-called *Biplot*. This plot displays both the original variables and the transformed observations on the principal components axes and factor loading axes, respectively. For an overview of this technique see Gower and Hand (1996). Now, when combining the results of fragmentation and DWA test methods, the diversity (non-homogeneity) in the latter set of variables becomes clearly visible in the Biplots. Figure 1 shows the Biplot of DWA test results combined with those of the LA test. Three groups of variables, pulling more or less in the same direction, become visible: the first group made of the 4 freeze/thaw tests plus MSS and Micro-Deval tests, the second group represented by the ST test and the third group formed by LA test and Porosity, both pulling in the same direction.



**Figure 1:** Principal Component Biplot (Numbers = Records)

Finally, the picture becomes complete when combining the data on 9 fragmentation variables and all 10 DWA variables (only Bg standard was excluded to avoid a singular data matrix). Figure 2 shows the loadings of these 19 variables on factors 1 and 2 as the result of a factor analysis after (varimax) rotation.



**Figure 2:** Loadings of Variables on Factors 1 and 2 (b,m,s,A,L,P as defined in Fig.1)

#### 4. CONCLUSIONS

Principal Component and Factor Analysis prove to be powerful techniques for comparing different test methods. One can clearly distinguish 3 different groups (represented by factors): the fragmentation test methods (TFVdry and TFVwet excluded) with high positive loadings on factor 1, the duration tests with high loadings on factor 2 (with EN 1367-1 and ST test “outlying” a little bit) and the third group made of the 10 % Fines Tests (TFVdry and TFVwet) with high negative loadings on factor 1.

The interrelationships between the test methods of different groups can be further studied through *Canonical Correlation Analysis (CCA)*. This technique can also be used to study relationships between the diverse DWA test methods. Moreover, CCA not only gives a comprehensive picture of the interrelations but also yields redundancy measures which may help to virtually derive test results from results based on related test methods. This way CCA may help to reduce laboratory efforts and costs.

#### REFERENCES

- PM-Techmodel 1.0: User Manual. Petromodel Ltd., Reykjavik 2002.
- Gower, J. C. and D.J. Hand: *Biplots*. Chapman & Hall, London 1996.
- Bjarnason, G., Erlingsson, S. and P. Petursson: *Aggregate Resistance to Fragmentation, Weathering and Abrasion*. BUSL Research Report E-37, Reykjavik 1999.
- Techmodel® is a registered trademark of Petromodel Ltd.